Turbulence Modelling

Theory and Practice of Modelling Turbulent Flow in Flotherm

Turbulence Modelling

- Equations for laminar flow apply equally to turbulent flows
- Impractical to solve High Reynolds Number flows
- Use Turbulence Modelling

Turbulence

- To model a turbulent flow, the temporal terms of the conservation equations would have to have a time step (dt) small enough to capture all turbulent fluctuations on even the smallest time scales.

Turbulence

- All physical dimensions of the control volume cells (dxi) terms would have to be as small as that known as the Kolmogarov scale, which decreases non-linearly with an increase in Reynolds number.

Turbulence

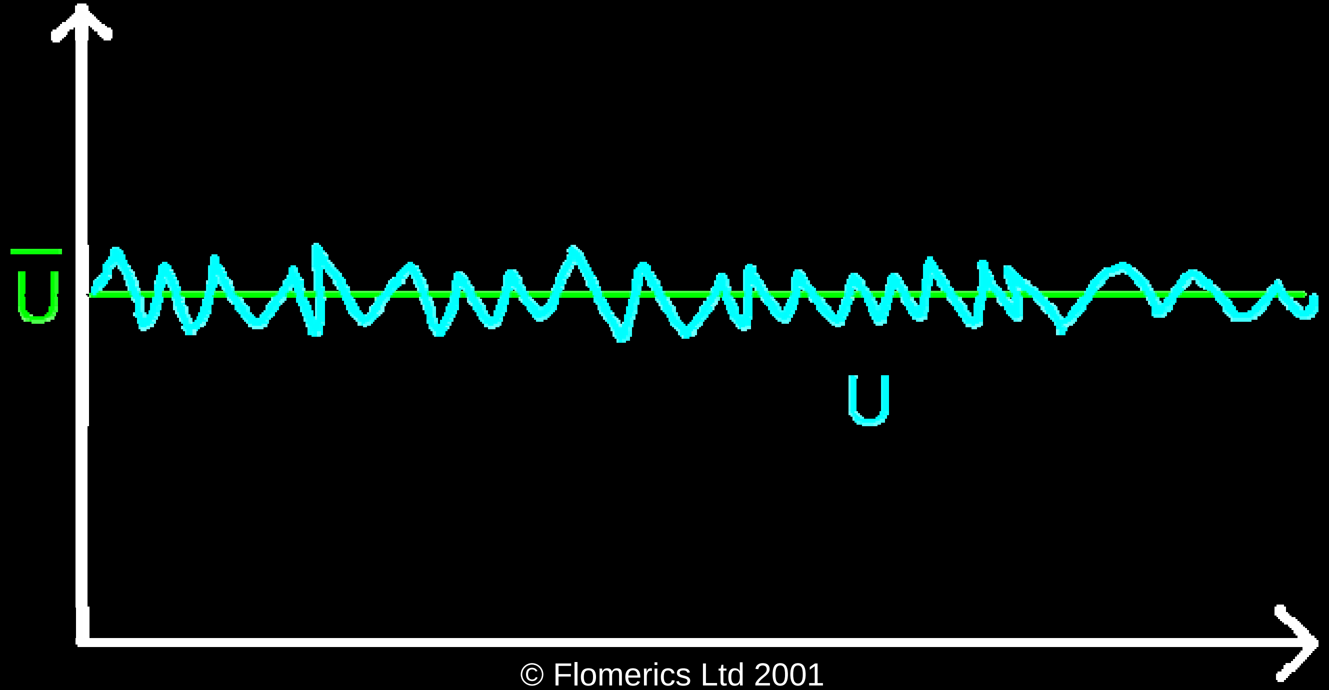
- Instead, treat flow as consisting of mean and fluctuating parts:

$$U = \overline{U} + u'$$

$$H = \overline{H} + h'$$

Turbulence

- Primarily, we are interested in the mean flow:



Turbulence

- Substitute back into the governing equations
- Gives extra terms called Reynolds stress (from the momentum equations) and Reynolds flux (from the energy equation)

Turbulence

- We could write equations for the Reynolds stresses and Reynolds Fluxes and solve them in conjunction with the other governing equations

- In FLOTHERM, we make use of simplifying assumptions

Turbulence Modelling

- The simplest view of the behaviour of a turbulent fluid is that it is just like that of a laminar one but with an increased viscosity
- This viscosity may vary from place to place.

Turbulence Modelling

- Physically, turbulence increases the mixing and heat transfer in the fluid
- We represent this as increased viscosity
- Increased thermal conductivity

Turbulence

- We can now write:

$$\mu_{\text{eff}} = \mu + \mu_t$$

- Where:
- μ = Laminar viscosity
- μ_t = Turbulent viscosity

Turbulence Models

- This concept is usually attributed to Boussinesq (1877) and is called the "EFFECTIVE-VISCOSITY HYPOTHESIS".
- One important implication is that turbulent shear stresses are proportional to velocity gradients.
- The hypothesis has been known for many years not to be valid in all circumstances; but it is often so close to the truth that it is very widely used.

Turbulence

- Turbulent fluids have an effective viscosity
- Prandtl expressed as follows:
- $\mu = \rho\, l^2 \,|\partial u/\partial y|$

Turbulence

- l is the mixing length
- Mixing length characterises the local structure of the turbulence
- May relate to the geometry of the system

Turbulence Models

- The main limitation imposed is that the eddy viscosity is the same in all directions at any point.
- This may not be true of turbulent viscosity, which is effectively a property of the flow.

Turbulence

- Turbulent conductivity is then evaluated from the following formula:

$$k_t = \frac{\mu_t C_p}{Pr_t}$$

- Pr_t is the Turbulent Prandtl Number.
- In FLOTHERM, $Pr_t = 0.9$

Models

- Turbulence models predict turbulence viscosity
- FLOTHERM has two forms of model:

- Zero equation model
- Two equation model

Zero Equation

- Turbulent viscosity depends on an algebraic function of flow properties and geometry

- FLOTHERM has:
- Automatic Algebraic Model (default)
- Revised Algebraic Model

Two Equation

- The two equation model uses two differential transport equations to predict the eddy viscosity on a cell by cell basis.
- FLOTHERM has:
- Standard KE Model
- Revised KE Model

Automatic Algebraic

- Based on LVEL model of Spalding et al
- Sets turbulent viscosity based on wall distance and local fluid velocity
- Turbulent viscosity varies from cell to cell in the bulk flow
- No cap on viscosity in bulk fluid

Automatic Algebraic

- We define:
- dimensionless velocity parallel to the wall, u^+
- dimensionless distance from the wall, y^+
- These are based on the wall shear stress and fluid properties

Automatic Algebraic

- Friction velocity

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- Distance

$$y^+ = \frac{\rho u_\tau y}{\mu}$$

- Velocity

$$u^+ = \frac{u}{u_\tau}$$

Automatic Algebraic

- Assume that there is a universal relationship between u$^+$ and y$^+$
- This is called the Law of the Wall
- Determine the effective viscosity

Automatic Algebraic

$$\mu^+ = \frac{\partial u^+}{\partial y^+}$$

- With

$$\mu^+ = \frac{\mu_{eff}}{\mu}$$

Automatic Algebraic

- The wall distance at each point in the solution domain is computed
- The Law of the Wall is applied to calculate the effective viscosity

Automatic Algebraic

- Model is ideal for system-level model
- It has been proven for more complex interactions of fluids and solids
- There is no limit on the viscosity
- Assumes fully developed flow

Revised Algebraic

- The revised algebraic turbulence model uses a single value of turbulent viscosity in the bulk of the fluid (i.e. outside of wall boundary layers).

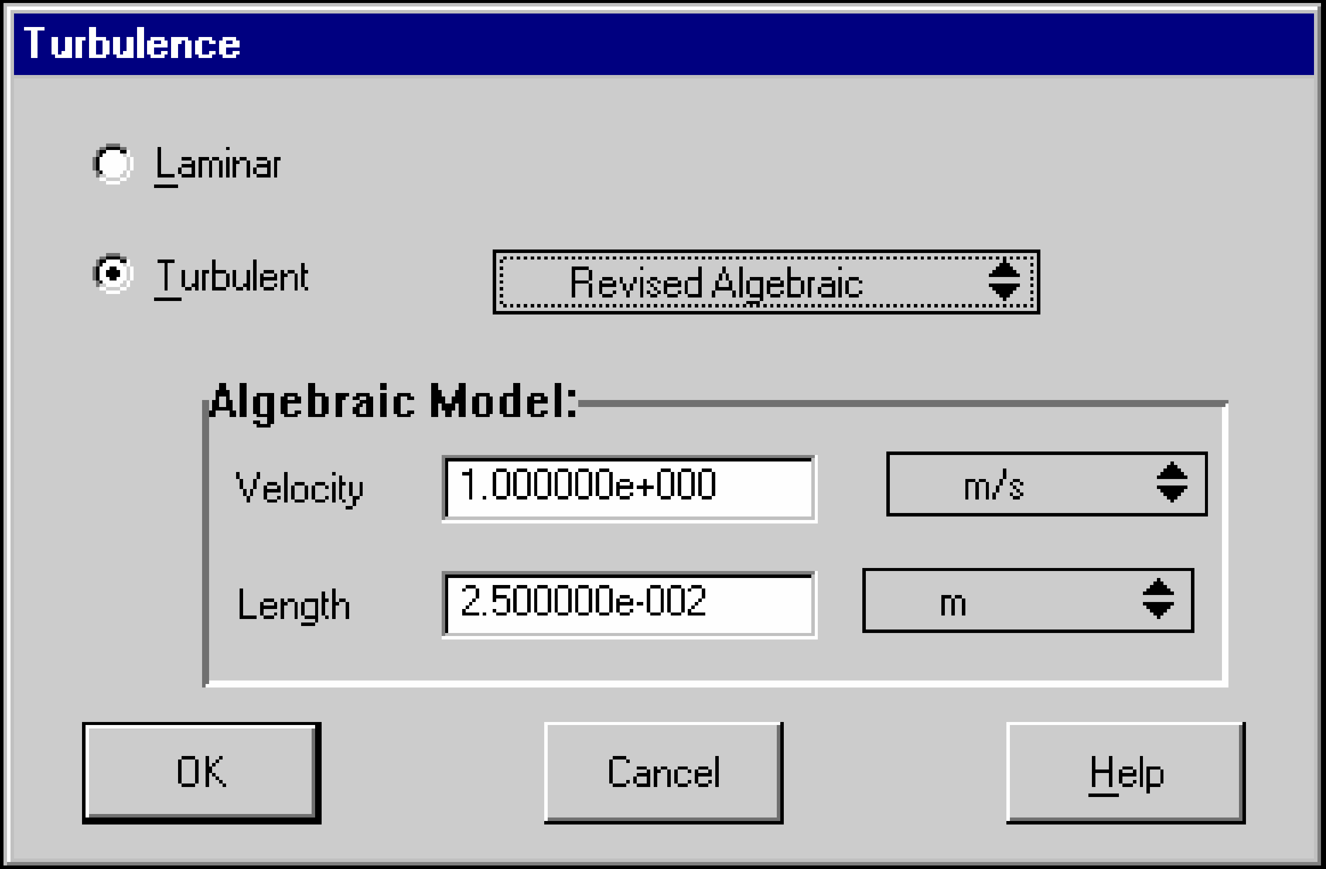
Revised Algebraic

- In the bulk fluid, the viscosity is given by:

$$\mu_t = 0.01\rho U d$$

- Where U and d are the user specified velocity and length

Revised Algebraic



○ Laminar

● Turbulent: Revised Algebraic

Algebraic Model:

Velocity: 1.000000e+000 m/s

Length: 2.500000e-002 m

Revised Algebraic

- For a channel flow, constant should be lower than the 0.01.
- If flow is simple channel flow, can set user specified velocity to 1/6 * channel velocity

Revised Algebraic

- Near the wall, the viscosity is modified

- Viscosity is reduced according to the Law of the Wall

Standard KE

- This turbulence model calculates two variables:
- the kinetic energy of turbulence (k)
- the dissipation rate of k (denoted ε).
- Physically

$$k = \sqrt{u'^2 + v'^2 + w'^2}$$

Standard KE

- Method proposed by Harlow and Nakayama (1968)
- Solve transport equations for k and ε
- Turbulent viscosity is given by:

$$\mu_t = 0.09\ \rho \frac{k^2}{\varepsilon}$$

Standard KE

- For steady, incompressible flow

$$\frac{\partial U_i k}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\left[\mu + \frac{\mu_t}{\sigma_k}\right]\frac{\partial k}{\partial x_i}\right) = P + G - \varepsilon$$

$$\frac{\partial U_i \varepsilon}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\left[\mu + \frac{\mu_t}{\sigma_\varepsilon}\right]\frac{\partial \varepsilon}{\partial x_i}\right) = C_1\frac{\varepsilon}{k}\left(P + C_3 G\right) - C_2\rho\frac{\varepsilon^2}{k}$$

Standard KE

- P is the production of k by shear forces

$$P = \mu_{eff} \frac{\partial U_j}{\partial x_i} \left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right)$$

- G is the production of k by buoyancy

$$G = \frac{\mu_{eff}}{\sigma_T} \beta g_i \frac{\partial T}{\partial x_i}$$

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Standard KE

- Constants used (derived empirically):

C_1	1.44
C_2	1.92
C_3	1.0
σ_k	1.0
σ_ε	1.217

Standard KE

- Stratification option for large enclosures with stable stratification
- Sets G = 0.0
- Limits mixing between adjacent cells

Standard KE

- The equations for k and ε are solved together with the other equations
- The results give values of viscosity that vary cell by cell throughout the domain

Standard KE

- The k-epsilon model is good for resolving flow in large enclosures
- Also good in domains with large velocity gradients
- Grid must be fine enough to resolve velocity gradients
- Two extra equations ⇒ Computationally expensive

Revised KE

- Revised KE incorporates a correction factor that reduces viscosity in near-wall cells
- BUT, it does not work with prisms in FLOTHERM
- Generally, best avoided unless you have good reason to use it

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Transitional Flow

- None of the turbulence models considers transitional flow

Separating Flow

- Separating flow is better handled by the KE models
- Separating flow is not handled well by algebraic models
- Important for impinging jets and for stagnation regions, if heat transfer in these regions dominates